Transmission Networks Resource Allocation Through Direct Mixed-integer AC Optimal Power Flow Applications with *Gurobi*

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***Abstract*-- As power systems develop, advanced analytical methods become crucial for efficient planning and operation. In this sense, the Optimal Power Flow (OPF) problem have evolved to incorporate, for example, discrete and stochastic variables–based models. As such, OPF became an effective tool for various power system-related studies, including contingency analyses, unit commitment or resource allocation. Also, combinatorial optimization problems, being both among the most challenging, on one hand, and commercially appealing, on the other, prompted the development of high-end commercial platforms like Gurobi. Hence, this work attempts to take advantage of the latest Gurobi Optimizer release features to directly address combinatorial AC OPF problems defined as non-convex mixed-integer quadratically constrained quadratic programming (MIQCQP). The study evaluates numerical performance on transmission network maximum peak capacity determination and optimal new generation allocation problems for a selection of available test cases (IEEE 30 and 118 bus systems), whereas transmission loss minimization is also targeted. Tests often yielded useful results in less than a few minutes, with effective combined optimization of active/reactive power flow and allocation of potential new generators to meet demand growth targets, among a set of candidate buses. However, constraints selection and parameters' fine-tuning impacted performance, sometimes leading into unappealing results.**

***Index Terms*-- AC optimal power flow; Gurobi Optimizer; transmission systems; hosting capacity; resource allocation.**

I. INTRODUCTION

As power systems expand in both size and complexity, deploying more sophisticated analytical methods and tools becomes critical for smooth operation and planning, concerning both technical and economic aspects. In that regard, the Optimal Power Flow (OPF) is an important technique, which originated as a derivation of the classical economic dispatch [1], whereas its current conceptualization trace back to [2] and [3].

An OPF, most essentially, is a numerical optimization problem characterized by introduction of the power flow equations as constraints, along with other variables and equations set to model electric network-related issues [4]. Mathematically, this can lead into a wide range of formulations, depending on the modeling approach adopted for each element considered. Traditionally, due to solution methods and processing power limitations, many OPF applications prioritized active power flow dynamics, and often depended on approximations, relaxations, decompositions and strictly continuous variables to ensure tractability [5]-[6]. Further, total dispatch cost minimization, given a set of transmission limits, were commonly taken as a reference. However, as the numerical optimization field progressed, many results were taken advantage of to develop more robust OPF frameworks.

Throughout its development, different classical [7]-[9] and modern meta-heuristics [8]-[10] optimization solution principals have been adapted to OPF-related applications, making the tackling of underling formulations including pronounced non-linarites, non-convexities, discreet or stochastic elements, multi-stage coupled variables, and so on, more appealing. Further, tractability of problems with increasingly greater dimensionality were achieved, also due to the evolution of microprocessors. As a consequence, the scope of OPF expanded, and it became an important technique for more complex analysis associated with power systems planning, expansion and operation, such as: unit commitment; hosting capacity; contingency and security; operational risk assessment; system design; optimal resource allocation, among others [11]-[12]. Regardless, even a more conventional AC OPF is still considered a high-complexity problem, given the pronounced non-linearity and non-convexity of coupled active and reactive AC power flow equations. Moreover, OPF the that absorb combinatorial optimization elements are among the most complex literature, recognized as high-end *NP-hard* [13].

Therefore, a vast academic literature encompassing OPF-related works have been developing over the years [14]. However, in the context of applied engineering, well-established optimization solvers and platforms are typically required for the implementation of embedded solutions, whether it be dedicated OPF software, or general-use numerical optimization toolsets, which depend on proper and compatible mathematical models inputs to be explicitly provided by the

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user [15]. Regardless, when those platforms are taken advantage of, computational performance can vary significantly depending on the synergy of the solver-formulation-hardware match-up, whereas each toolset is based on different underling solution methods and problem classes.

Furthermore, when it comes to combinatorial optimization, those are, in one hand, among the hardest known problems, mainly due to the complexities arisen by combinatorial explosion [16]. On the other hand, they have great commercial appeal, due its potential impacts in logistics, routing and operations research, prompting the development of high-end commercial solvers (e.g., *Gurobi*, CPLEX, XPRESS) capable of providing significantly better performance than available alternatives, especially for mixed-inter problems [17]-[21]. This can be a deciding factor when evaluating the feasibility of a potential application. Historically, the best results obtained with such platforms are for strictly defined sets of mathematical formulations, such as linear programming (LP), mixed-integer linear programming (MILP) or even mixed-inter quadratic programming (MIQP), whereas, for the latter, considerations regarding non-convexities in the constrains and/or objective functions can further limit applications [22].

In this context, the latest major *Gurobi Optimizer* release (version 11, launched late 2023) [23] brings a promising potential to deal with non-convex mixed-integer quadratically constrained (MIQCP) and quadratic programming (MIQCQP) problems, which, in turn, can become a convenient pathway to achieve direct solution of combinatorial AC OPF (i.e., full blown AC OPF can be non-convex quadratic if the AC power flow equations are in rectangular form). For example, resource allocation or even unit commitment could be directly and precisely tackled through robust MIQCQP formulations to integrate discrete decision making with AC OPF, if the computation performance proves to be sufficiently appealing.

With all the discussed, the authors couldn't find works specifically evaluating the potential of *Gurobi Optimizer* applications to address combinatorial AC OPF problems. In this sense, this work seeks to accomplish as much, whereas problems associated with transmission network maximum load capacity determination and optimal new generation allocation (also considering transmission losses minimization under peak demand conditions) are taken as references. More specifically, main contributions can be highlighted as: *i*) systematic detailing of mathematical formulations compatible with *Gurobi Optimizer* applications, considering combinatorial AC OPF problems defined as MIQCP/MIQCQP; *ii*) numerical evaluation of the solution approach for some test cases taken as references (standard IEEE 30 and 118 bus systems).

The remaining of the paper is as follow: section II presents a summary of relevant topics for the formulation and solution of OPF; section III details the problems focused on, regarding its statement, formulation and solution framework; section IV discusses the numerical results of examples for different test systems and scenarios evaluated; and section V concludes the document with discussions and suggestions for future developments.

II. Optimal Power Flow Highlights

A. *Essential Concepts and Formulations*

The OPF consists of a numerical optimization problem. Regarding its most essential features, it can be characterized generally as (1)-(3), as typically found in the literature [5]-[8].

$$\min_{x,u} f(\boldsymbol{x},\boldsymbol{u}) \tag{1}$$

s.t.:

$$h_i(\boldsymbol{x},\boldsymbol{u}) = 0, i \in \mathcal{E} \tag{2}$$

$$g_j(\boldsymbol{x},\boldsymbol{u}) \leq 0, j \in \mathcal{I} \tag{3}$$

Equations (1), (2) and (3) correspond, respectively, to the objective function, the *i*-th equality constraint ($\mathcal{E}$), and the *j*-th inequality constraint ($\mathcal{I}$), all written as a function of $\boldsymbol{x}$ and $\boldsymbol{u}$, the state and control variable arrays, respectively. Note that $\boldsymbol{x}$ and $\boldsymbol{u}$ encompass subsets of the electric variables typically associated with standard power flow problems, that is, the buses' complex voltage components and generation/load active/reactive power balances for the corresponding network.

Regarding the objective function, different formulations can be applied depending on the operational priorities. Common targets include minimizing dispatch costs, transmission losses, or voltage deviations, all of which are formulated as functions of $\boldsymbol{x}$ and $\boldsymbol{u}$. Multi-objective functions can also be used.

More specifically, the set of constraints $\mathcal{E}$ usually encapsulates the active and reactive power balance equations at each bus, presented in their rectangular form in (4) and (5), respectively, where: $\boldsymbol{e_k}$ e $\boldsymbol{f_k}$ are variables for the real and imaginary components of the complex voltage at bus *k*; $\boldsymbol{P_k^g}$, $\boldsymbol{Q_k^g}$, $\boldsymbol{P_k^l}$ e $\boldsymbol{Q_k^l}$ are the variables for active (*P)* and reactive (*Q*) power being generated (*g*) or consumed (*l*) at bus *k*; *N* is the number of buses in the system; *m* is an index associated with the buses adjacent to bus *k*; and G_{km}e B_{km} are parameters associated with the element of index *km* in the conductance and susceptance bus matrices. Note that many different simplified balance equations can be applied instead of (4)-(5), which are sometimes is prioritized to ensure tractability [24].

$$\boldsymbol{P_k^g} - \boldsymbol{P_k^l} = \sum_{m=1}^{N} \{\boldsymbol{e_k}(G_{km}\boldsymbol{e_m} - B_{km}\boldsymbol{f_m}) + \boldsymbol{f_k}(G_{km}\boldsymbol{f_m} - B_{km}\boldsymbol{e_m})\}, k\epsilon\{1,\dots,N\} \tag{4}$$

$$\boldsymbol{Q_k^g} - \boldsymbol{Q_k^l} = \sum_{m=1}^{N} \{\boldsymbol{f_k}(G_{km}\boldsymbol{e_m} - B_{km}\boldsymbol{f_m}) - \boldsymbol{e_k}(G_{km}\boldsymbol{f_m} + B_{km}\boldsymbol{e_m})\}, k\epsilon\{1,\dots,N\} \tag{5}$$

In turn, the set of constraints $\mathcal{I}$ usually includes the desired operational limits for the transmission network, such as: limits on active (6) or reactive (7) power injection; buses' voltage magnitudes (8); line loading limits (9); and limits on transformer taps (10). $|\boldsymbol{V_k}|$, $|\boldsymbol{I_{km}}|$ e $\boldsymbol{T_{km}}$ correspond to: voltage magnitude at bus *k;* tcurrent magnitude flowing from bus *k* to *m*; and transformer tap between buses *k* and *m*, respectively. The parameters on the left and right sides of each inequality in (6)-(10) are the minimum/maximum limits for each variable.

$$P_k^{g,lMin} \leq \boldsymbol{P_k^{g,l}} \leq P_k^{g,lMax}, k\epsilon\{1,\dots,N\} \quad (6)$$

$$Q_k^{g,lMin} \leq \boldsymbol{Q_k^{g,l}} \leq Q_k^{g,lMax}, k\epsilon\{1,\dots,N\} \quad (7)$$

$$\left|V_k^{Min}\right| \leq |\boldsymbol{V_k}| \leq \left|V_k^{Max}\right|, k\epsilon\{1,\dots,N\} \quad (8)$$

$$I_{km}^{Min} \leq |\boldsymbol{I_{km}}| \leq P_{km}^{Max}, k,m\epsilon\{1,\dots,N\} \quad (9)$$

$$T_{km}^{Min} \leq \boldsymbol{T_{km}} \leq Q_{km}^{Max}, k,m\epsilon\{1,\dots,N\} \quad (10)$$

B. Known Developments and Applications

Formulations derived form (1)-(10) result in strongly non-convex Nonlinear Programming (NLP) problems which, given (4)-(5), can be treated as non-convex Quadratically Constrained Quadratic Programming (QCQP) problems, if the remaining constraints are adequately introduced. As mentioned, it is always possible to adopt relaxations or approximations, which can simplify the resulting formulation, usually at the cost of lower accuracy or scope. Furthermore, proper treatment of some issues inherent to power system operation require the introduction of integer/binary variables, which yields high-complexity Mixed-Integer NLP (MINLP) problems. As an example, an unit commitment-oriented AC OPF can couple binary variables representing dispatchable units ON/OFF states – $\boldsymbol{u_k}$ with (4)-(5), which substantially increases the resulting formulation complexity. Note that, in such cases, multiplying the parametric limits in (6)-(7) by $\boldsymbol{u_k}$ is a common modeling practice to ensure that generation equals zero only if $\boldsymbol{u_k} = 0$, and, otherwise, that (6)-(7) maintain its original effect [25].

This work focuses on combinatorial AC OPF, whereas the issues of transmission network maximum load capacity determination and optimal new generation allocation are adopted as a reference for evaluation of the Gurobi Opmizier toolset. However, many power system-related problems that can be addressed through OPF approaches have been explored. These can include other dimensions of the system in the modeling, beyond the electrical/power flow-related dynamics, such as generators' production or cost functions, in addition to possible definition of deterministic or stochastic problems with multiple time stages. Moreover, even for well-defined problems (e.g., unit commitment, reactive power dispatch), different modeling approaches can be adopted, which, in turn, can fall into different classes of optimization problems that could be tackled through a diverse set of underlying solution principles and/or numerical platforms. To provide a better contextualization, Table I summarizes recent OPF-related works exploring optimization solvers ready application.

Inspection of Table I suggests that, although adequate to achieve encompassing and high-precision AC OPF, direct fully non-convex MIQCP/MIQCQP applications are yet not commonly found, especially when the main focus lays on the discrete decision-making dimensions of the underlying problem. This is due to the high-complexity of this formulation category, which can often lead into instabilities or prohibiting computational burdens. Furthermore, widely available solvers, be it commercial or not, are often incompatible with fully non-convex MIQCP/MIQCQP, or can only tackle it under some specific conditions (e.g. CPLEX currently does not accept equality constraints in MIQCP problems [39]). Even when fully compatible, they can still demonstrate unappealing performance [40] for applications with real-life systems.

TABLE I
SAMPLE OF OPF APPLICATIONS IN THE LITERATURE BASED ON COMMERCIAL/FREE SOLVERS

Solver	Open-Source?	Compatibility	Applications Examples
Gurobi	x	LP, QP, QCQP, NLP, SOCP, MIP, MILP, MIQP, MIQCQP, MISOCP, MINLP.	▪ Convex OPF formulation through linearization of the power-voltage hyperbolic relationship; applies MILP problems [26] ▪ Radial distribution networks power flow optimization, considering the insertion of renewable resources; applies second-order cone programming (SOCP) problems [27] ▪ OPF with active constraint identification based on deep convolutional neural networks; applies LP problems [28] ▪ Reactive power flow optimization problems using a mixed-integer SOCP (MISOCP) approach; based on the combined application of Gurobi, IPOPT, and Mosek [29].
CPLEX	x	LP, QP, QCQP, NLP, SOCP, MIP, MILP, MIQP, MIQCQP, MISOCP.	▪ Comparison of MIQCQP and MILP formulations for nodal voltage analysis in distribution systems [30] ▪ Economic dispatch optimization based on minimization of: transmission losses (applying QCP), fuel usage (applying MIQP), and the valve point effect [31]
FICO XPRESS	x	LP, QP, QCQP, NLP, SOCP, MIP, MILP, MIQP, MIQCQP, MISOCP, MINLP.	▪ Distribution networks losses minimization, using MIQCQP problems; compares different solvers like FICO Xpress and CPLEX [32] ▪ Comparison of linearized AC models for power flow optimization; applies MILP [33].
lp_solve	✓	LP, MIP, MILP, MIQP.	▪ Minimization of renewable generation costs, with or without transmission losses, through optimal current flow analysis; applies MILP problems [34] ▪ Optimization of e-vehicle charging, for maximizing total amount vehicles of charged vehicles for the lowest cost; applies using MILP problems [35].
GLPK	✓	LP, MIP, MILP.	▪ Optimized sizing of combined cooling, heating, and power systems, considering a deterministic load scenario; applies MILP problems [36].
IPOPT	✓	NLP	▪ Calculation of AC OPF using an (non-linear programming) NLP framework, with the objective aimed at total system supply cost minimization [37]
MOSEK	x	LP, QP, QCQP, SOCP, SDP, MIP, MILP, MIQP, MIQCQP, MISOCP.	▪ Development of an optimization proxy for AC OPF calculation, employing SOCP [38].

III. Problems Statement and Solution Framework

Expanding total installed capacity as demand is projected to grow is a major concern in power system planning [41], whereas a network's capability of effectively suppling all loads also depends on transmission efficiency, topology and resources placement. This is greatly influenced by the relative allocation between power sources and loads, and the flow profile (e.g. bus voltages profiles). Traditionally, large-scale generation facilities (i.e., several hundreds to thousands of megawatts) have been implemented far from the loads centers, sometimes, due to geological or environmental constraints. Moreover, as energy resources more easily explored become rarer, and loads concentrate in densely populated areas, large-scale enterprises are driven further away from the consumption ends, sometimes eliciting the development of new transmission technologies, like HVDC [42]. As an alternative, approaches based on high penetration of smaller generation sites, have been drawing attention [43]. Regardless, optimized placement of new resources, be it transmission lines or generation, not only can improve the systems' capacity, but also reduce technical or financial requirements needed to achieve a certain level of growth. Nevertheless, this can implicate in the solution of highly complex optimization problems, as mentioned.

Hence, this work uses the optimized allocation of new generating sites in previously existing transmission networks to conceive combinatorial AC OPF problems as the basis for the targeted applications with *Gurobi Optimizer*. More specifically, the problems defined are aimed at: *i*) given a power system with a fixed grid topology, determine the maximum peak demand it can accommodate, through the optimized distribution of active/reactive dispatch among a combination of both existing generators and an undetermined set of potential new sites; *ii*) optimally allocate the new generation set over connections points that lead into the minimization of transmission losses for the maximum demand growth attainable with respect to the baseline. The following topics present, respectively, the overall solution framework used, and specifies the mathematical formulations. Details regarding the premises adopted to enable numerical simulations will be discussed in the next section.

A. Solution Framework

The diagram in Fig. 1 illustrates the underlying solution framework applied, which is based on two problems. Both of them are stated similarly, but each focuses on one specific objective. Firstly, an AC OPF model is derived from a standard power flow baseline case, whereas consideration for potential connections of new generation sites among a set of previously discriminated candidate buses is introduced. This results in a MIQCP formulation. This first problem focuses on estimating the maximum load burden supported by the network, which is achieved by multiplying all baseline active/reactive demands by a single gain (***DG***), and maximizing this gain (i.e., uniform growth is considered). Typical OPF constraints are applied, whereas binary variables are used to model the introduction (or not) of new generation capabilities on selected candidate buses. Power factor (PFC) limits can be added, whereas, for those, and for (8)-(9) like constraints, equations must be arranged to remain quadratic and compatible with the solvers' inputs.

In a following step, another AC OPF model is derived considering the same baseline circuit parameters, but all demands are now fixed to the values attained through ***DG*** maximization. Otherwise, this problem is similar to the first, except the objective function is set to be the minimization of total transmission losses, which yields an MIQCQP problem. Its solution is meant to rearrange the initial optimal allocation of the new generation sites, whereas the optimal active/reactive dispatch for a given configuration is treated implicitly to ensure the maximum demand target settings are fulfilled.

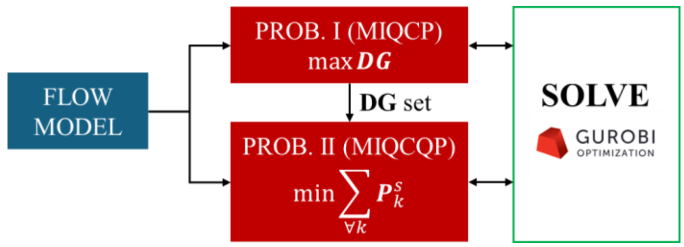

Fig. 1. Reference solution framework adopted

For tackling each problem in Fig. 1, *Gurobi Optimizer* was deployed, which is currently one of the most prominent optimization solvers in the world [23]. It was originally focused on large-scale linear programming (LP) and mixed-integer programming (MIP) problems. Since its inception, substantial improvements for these classes of problem were achieved [23], as well as an expansion in the range of tractable formulations. Its most recent major release (version 11, form November 2023; other minor versions have followed), the product supplier suggests that convex and non-convex problems with increasing levels of complexity can be solved reliably and with appealing performance, including for the MIQCQP type [44], which, as shown following formulations, include full blown combinatorial AC OPF applications. The platform has different programming interfaces in C++, Python, MATLAB, and others.

B. Mathematical Formulations

The MIQCP problem associated with the first step indicated in Fig. 1 is given by (11)-(36), where variables are in bold.

$$\max \boldsymbol{DG} \tag{11}$$

s.t.:

$$DG^{Min} \leq \boldsymbol{DG} \leq DG^{Max} \tag{12}$$

$$E^{Min} \leq \boldsymbol{e}_k \leq E^{Max}, \forall k \tag{13}$$

$$F^{Min} \leq \boldsymbol{f}_k \leq F^{Max}, \forall k \tag{14}$$

$$P_k^{Min} \leq \boldsymbol{P}_k^s \leq P_k^{Max}, \forall k \tag{15}$$

$$Q_k^{Min} \leq \boldsymbol{Q}_k^s \leq Q_k^{Max}, \forall k \tag{16}$$

$$\left|V_k^{Min}\right|^2 \leq (\boldsymbol{e}_k)^2 + (\boldsymbol{f}_k)^2 \leq |V_k^{Max}|^2, \forall k \tag{17}$$

$$(\boldsymbol{P}_k^s)^2 \geq PFC_k^{Lim}\,[(\boldsymbol{P}_k^s)^2 + (\boldsymbol{Q}_k^s)^2], \forall k \tag{18}$$

$$[(\boldsymbol{G}_{km})^2 + (\boldsymbol{B}_{km})^2] \cdot [(\boldsymbol{e}_k - \boldsymbol{e}_m)^2 + (\boldsymbol{f}_k - \boldsymbol{f}_m)^2] \leq \left|I_{km}^{Lim}\right|^2, \forall k, \forall m \quad (19)$$

$$\boldsymbol{P}_k^s = \sum_{m=1}^{N} [\boldsymbol{e_k}(G_{km}\boldsymbol{e_m} - B_{km}\boldsymbol{f_m}) + \boldsymbol{f_k}(G_{km}\boldsymbol{f_m} - B_{km}\boldsymbol{e_m})], \forall k \quad (20)$$

$$\boldsymbol{Q}_k^s = \sum_{m=1}^{N} [\boldsymbol{f_k}(G_{km}\boldsymbol{e_m} - B_{km}\boldsymbol{f_m}) - \boldsymbol{e_k}(G_{km}\boldsymbol{f_m} + B_{km}\boldsymbol{e_m})], \forall k \quad (21)$$

$$\boldsymbol{e}_k = E_{slk}, \forall k \epsilon \Omega_{slk} \quad (22)$$

$$\boldsymbol{f}_k = F_{slk}, \forall k \epsilon \Omega_{slk} \quad (23)$$

$$(\boldsymbol{e}_k)^2 + (\boldsymbol{f}_k)^2 = |V_k^{PV}|^2, \forall k \epsilon \Omega_{PV} \quad (24)$$

$$\boldsymbol{P}_k^s = \boldsymbol{P}_k^g - \boldsymbol{DG} \cdot P_k^{Load}, \forall k \epsilon \Omega_{PV}, \forall k \epsilon \Omega_{PQ}^{gen} \quad (25)$$

$$\boldsymbol{Q}_k^s = \boldsymbol{Q}_k^g - \boldsymbol{DG} \cdot Q_k^{Load}, \forall k \epsilon \Omega_{PV}, \forall k \epsilon \Omega_{PQ}^{gen} \quad (26)$$

$$P_k^{GenMin} \leq \boldsymbol{P}_k^g \leq P_k^{GenMin} \quad (27)$$

$$Q_k^{GenMin} \leq \boldsymbol{Q}_k^g \leq Q_k^{GenMin} \quad (28)$$

$$\boldsymbol{P}_k^s \leq \boldsymbol{u}_k P_k^{CBmax} - \boldsymbol{DG} \cdot P_k^{Load}, \forall k \epsilon \Omega_{CB} \quad (29)$$

$$\boldsymbol{P}_k^s \geq \boldsymbol{u}_k P_k^{CBmin} - \boldsymbol{DG} \cdot P_k^{Load}, \forall k \epsilon \Omega_{CB} \quad (30)$$

$$\boldsymbol{Q}_k^s \leq \boldsymbol{u}_k Q_k^{CBmax} - \boldsymbol{DG} \cdot Q_k^{Load}, \forall k \epsilon \Omega_{CB} \quad (31)$$

$$\boldsymbol{Q}_k^s \geq \boldsymbol{u}_k Q_k^{CBmin} - \boldsymbol{DG} \cdot Q_k^{Load}, \forall k \epsilon \Omega_{CB} \quad (32)$$

$$\sum_{k \in \Omega_{CB}} \boldsymbol{u}_k \leq NU^{Lim} \quad (33)$$

$$\boldsymbol{u}_k \in \{0,1\}, \forall k \epsilon \Omega_{CB} \quad (34)$$

$$\boldsymbol{P}_k^s = -\boldsymbol{DG} \cdot P_k^{Load}, \forall k \epsilon \Omega_{PQ}^{load} \quad (35)$$

$$\boldsymbol{Q}_k^s = -\boldsymbol{DG} \cdot Q_k^{Load}, \forall k \epsilon \Omega_{PQ}^{load} \quad (36)$$

Where:

- k and m are indexes associated with a given bus (k) or a bus adjacent to it (m).
- $\boldsymbol{DG}$ is the gain modeling a uniform demand growth.
- DG^{Max} and DG^{Min} are $\boldsymbol{DG}$ upper / lower boundaries.
- $\boldsymbol{e}_k$, $\boldsymbol{f}_k$ are real / imaginary voltage components at bus k (*pu*).
- E^{Max}, E^{Min}, F^{Max}, and F^{Min} are upper / lower boundaries of $\boldsymbol{e}_k$ / $\boldsymbol{f}_k$ at each bus k (*pu*)
- $\boldsymbol{P}_k^s$ and $\boldsymbol{Q}_k^s$ are the active / reactive power balance (i.e., injected generation minus drawn load) at bus k (*pu*)
- P_k^{Max}, P_k^{Min}, Q_k^{Max}, and Q_k^{Min} are upper / lower boundaries of $\boldsymbol{P}_k^s$ / $\boldsymbol{Q}_k^s$ for each bus k (*pu*).
- $|V_k^{Max}|$ and $\left|V_k^{Min}\right|$ are the upper / lower boundaries of the voltage magnitude at bus k (*pu*).
- PFC_k^{Lim} is the PFC limit set at bus k.
- $\left|I_{km}^{Lim}\right|$ is the maximal current magnitude allowed to flow between buses k and m (*pu*).
- N is the total number of buses in the network.
- G_{km} and B_{km} are element of index km in the conductance (G) / susceptance (B) bus matrices.
- Ω_{slk} is the set of indexes associated with slack buses.
- Ω_{PV} is the set of indexes associated with PV buses.
- Ω_{CB} is the set of for candidate buses, (i.e., possibility for new generation connections).
- Ω_{PQ}^{gen} is the set of indexes associated with PQ buses connected to generators.
- Ω_{PQ}^{load} is the set of indexes associated with PQ buses connected to passive loads only.
- $\boldsymbol{P}_k^g$ and $\boldsymbol{Q}_k^g$ are dispatched active / reactive power of a previously existing resources connected to bus k (*pu*).
- P_k^{GenMax}, P_k^{GenMin}, Q_k^{GenMax}, and Q_k^{GenMin} are upper / lower boundaries for active / reactive power dispatches of existing resources at bus k (*pu*).
- $\boldsymbol{u}_k$ is a binary variable representing if new generation was connected to bus k ($\boldsymbol{u}_k$ = 0 means no).
- P_k^{CBmax}, P_k^{CBmin}, Q_k^{CBmax}, and Q_k^{CBmin} are upper / lower boundaries for active / reactive power dispatch new generation connected to bus k (*pu*), when $\boldsymbol{u}_k$ = 1.
- NU^{Lim} is the maximum total amount of new connections allowed.
- P_k^{Load} , and Q_k^{Load} are parameters for load drawn at bus k (*pu*), according to the baseline case.

The objective function is given by (11), targeting maximization of overall demand supported by the network. As for the constraints, (12)-(16) are the main variables boundaries, whereas (17)-(19) are operational limits regarding: acceptable voltage magnitude for each bus (17), buses' equivalent PFC limits (18), and current magnitude limits through the branches (19). Those constrains don't use approximations, but were formulated so the resulting equations are in rectangular form, quadratic and containing no fractions with variables on the denominator, to ensure compatibility with the solver. Moreover, (20)-(21) correspond to the active and reactive power balance equations (with power flow equations in rectangular form), whereas (22)-(24) fix voltage values at controlled buses, that is, the real and imaginary components at the slack bus (22)-(23) and the magnitude at PV buses (24). Furthermore, (25) couple active power flow equations in (20) to the generation/load balance in PV buses and PQ buses connected to generators. Complementary, (26) does the same in the case of reactive power. Additionally, (27)-(28) represent the possible dispatch ranges for buses with existing resources (PV or PQ). Further, (29)-(32) couple active / reactive power flow equations in (20)-(21) to the generation / load balance in buses where there could be new generation or not, whereas (33) limits the total amount of new sites allowed, and (34) indicates that binary variables represent the statuses (i.e., new connection / no new connection) at candidate buses. Finally, (35)-(36) couple active / reactive power flow equations in (20)-(21) with the load balance in strict PQ buses (this include transfer buses, in which power drawn in set to zero).

In turn, the MIQCQP problem associated with the follow-up step in Fig. 1 is given by (37)-(39). Note that most variables and constraints are similar to the presented before, the main difference being that $\boldsymbol{DG}$ is reduced to a parameter set by the results of the first problem. Therefore, (38) simply highlights the constraints that remain identical to the first step, whereas (39) highlights the ones in which similar equations are applied, except for the value of $\boldsymbol{DG}$ being fixed. Regarding the objective function, it is set up to compute the total transmission losses precisely, as the residual sum of the AC power flow equations for all buses. This results in a challenging formulation, suited to evaluate the solver's robustness.

$$\min \sum_{\forall k} \boldsymbol{P}_k^s \tag{37}$$

s.t.:

$$(13)\text{-}(24),\ (27)\text{-}(28),\ (33) \tag{38}$$

$$(25)\text{-}(26),\ (29)\text{-}(32),\ (34)\text{-}(35) \tag{39}$$

IV. Numerical Evaluations

The problems discussed in Section III were implemented and tested with the latest minor release of the *Gurobi* solver (11.0.3, July 2024) [44], under an academic license. The Python API was adopted as the modeling interface, with the aid of Spyder IDE 5.4.3 (Python 3.11) running within an Anaconda 2.5.4 environment. All simulations were executed on a Windows 10, Intel Core i5-9400 CPU@2.90GHz/8.00GB RAM computer. Regarding the test beds, the IEEE 30 bus and 118 bus standard cases were used, whereas the circuit and baseline power flow data were derived from Matpower 8.0, which is refereed to for detailing of the model's parameters [45]. Nevertheless, as some of the information necessary to fully set up the targeted problems was unclear, incomplete or unavailable, premises were adopted to enable the applications.

The following topics detail each case and discuss the results obtained separately, however, some considerations were similar, such as: *i*) to prevent unrealistic results, existing generators maximum dispatch capacities are estimated to values relative to their baseline case; *ii*) the potential introduction of new generation sites privileges small-scale resources (only one new facility is permitted per bus); *iii*) only busses with some load and no previously existing generation are regarded as candidates; *iv*) the maximum running time for optimization was set to 200s; vi) the gap between lower and upper objective bonds, which is used as a precision/convergence criterion to terminate execution, was set be lesser then 10^{-2}.

A. 30 Bus Case Set up and Results

In the IEEE 30 bus baseline case, there are 5 PV buses, all characterized by generating units, and the slack bus, whereas 18 buses were set as candidates. In the standard power flow baseline, the dispatches range, approximately, from 20MW to 60MW, with buses' equivalent PFC ranging from 0.48 to 0.96. Hence, existing generators active power limits are set between 1.5x and 0.2x of its baseline (except for the slack bus, set between 2.0x and 0.0x), with the reactive power limits being adjusted so that the PFC equivalent in the PV buses, at maximum generation, could be accommodated within the baseline range. In turn, new units, when added, are set to dispatch between 5MW and 30MW, with reactive power limits being -18Mvar and 18Mvar (i.e., regardless of dispatch, it was initially considered the resulting PFC equivalent in these units' connection buses should also be above 0.8, inductive or capacity). Voltage magnitudes in all buses are limited between 0.95pu and 1.05pu, with voltage controlled busses fixed at baseline values. All branches' current magnitudes limits are set to 1.0pu. For transfer buses, PFC and power balance limits parameters are all set to zero. Initial active / reactive loads parameters are all identical to the baseline case.

It is important to observe that these considerations are, in some measure, arbitrary, and could artificially curtail of inflate the network transmission capacity, including due to excessive constraining of reactive power flow ranges. However, given this study prioritizes the evaluation of the adequacy of the solution toolset for the given application, this should not substantially impact the nature of the analysis realized.

Therefore, a series of (11)-(36) type problems are initially solved, considering increasingly higher values for the NU^{Lim} parameter (33), and $1.0 \le \boldsymbol{DG} \le 5.0$, mainly to observe: maximum demand gain progression as more units are added, and the processing time progression as the combinatorial search space is altered by the reparametrizing of (33). The optimality gap after a solution is found or when the time limit expires are also noted. Fig. 2 summarizes those results.

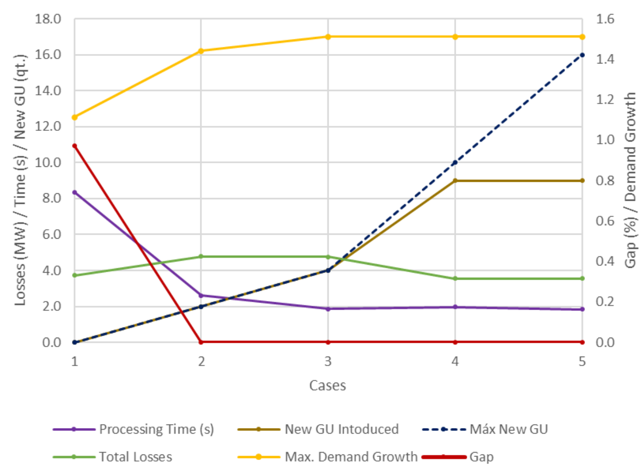

Fig. 2. $\boldsymbol{DG}$ maximizing in the 30 bus case

The results highlighted in Fig. 2 illustrate the highest processing time to solve a problem in this set was under 10s (which occurred with $NU^{Lim} = 0$, meaning only previously existing generator's dispatches are optimized) with most gaps reaching approximately 0.0%. Naturally, the resulting formulation for a 30 bus system, despite being a combinatorial AC OPF, is relatively small, totaling only few hundred variables and constraints. Furthermore, it can be noted that, as the NU^{Lim} is relaxed, the maximum ***DG*** and the total amount of new generation allocated increase, whereas the processing time was reduced. Nevertheless, all of them reached a plateau without further allocation of new resources. That is, even for NU^{Lim} = 18, the solver only chose to add 9 new units. In turn, the maximum demand gain sopped progressing with only 4 new connections, reaching ***DG*** = 1.51.

Therefore, the follow up problem (see Fig. 1) is set up to minimize transmission losses for ***DG*** fixed at 1.51x, whereas NU^{Lim} = 4 (i.e., the minimum amount of new resources allocated to achieve ***DG*** = 1.51 in the first problem). This yielded 4 new generators connections, totaling 2.83MW in transmission losses. Also, total processing time was 1.93s and the gap approximately 0.0%. Note that, in the initial problem, since transmission losses are disregarded, a different candidate bus set is chosen totaling 4.76MW in losses, although 4 new generators are also added (when NU^{Lim} = 4). Table II compares resource allocation in each case. Note that both the buses chosen to connect new recourses and the dispatch distribution at maximum demand conditions are significantly different.

TABLE II
TRANSMISSION LOSS MINIMIZATION FOR THE 30 BUS CASE (BASE IS 100MW)

Bus N° (Initial)	P_g/Q_g (pu)	Bus N° (Final)	P_g/Q_g (pu)
1 (*Slk*)	0.46 / -0.15	1 (*Slk*)	0.11 / 0.02
2 (*PV*)	0.75 / 0.35	2 (*PV*)	0.64 / 0.28
13 (*PV*)	0.30 / 0.13	13 (*PV*)	0.48 / 0.11
22 (*PV*)	0.32 / 0.40	22 (*PV*)	0.32 / 0.40
23 (*PV*)	0.16 / 0.11	23 (*PV*)	0.06 / 0.02
27 (*PV*)	0.18 / 0.13	27 (*PV*)	0.36 / 0.10
3* (*New*)	0.30 / 0.18	7** (*New*)	0.30 / 0.18
4* (*New*)	0.30 / 0.16	8 (*New*)	0.09 / 0.18
8 (*New*)	0.09 / 0.18	15** (*New*)	0.30 / 0.15
24 (*New*)	0.05 / 0.16	24 (*New*)	0.23 / 0.14

It is important to highlight that the maximum demand capacity observed was, primordially, a consequence of transmission constraints, especially with respect to the buses voltage magnitudes. This indicates that further increase could be achieved if more encompassing reactive power flow optimization measures were considered. One convenient way to evaluate that within the simulation framework developed is to loosen the PFC and reactive power boundaries associated with newly allocated resources and its buses.

Therefore, to complement the analysis, several simulations analogous to the ones which results are presented in Fig. 2 were performed for different PFC_k^{Lim} (18), Q_k^{CBmax} (31) and Q_k^{CBmin} (32) values. In general, those tests presented similar characteristics to the initially observed, regarding processing time, convergent optimality gaps and the plateaus for ***DG*** increase and maximum new connections allocated. An overall maximum ***DG*** of approximately 1.75 was achieved, for $PF_k^{Lim} \neq 0$ and $Q_k^{CBmax/min}$ magnitudes set to 30Mvar (in this case, NU^{Lim} = 7 was associated with ***DG*** reaching a plateau).

However, a few comments are important. Firstly, in some cases, the solver failed to resolve the MIQCQP problems associated with the follow up step, and transmission losses optimization was not achieved. This is possibly related to the heightening of the numerical instability of (37) for scenarios in which more complex reactive power flow set ups can be explored. Moreover, when $PF_k^{Lim} = 0$ in the MIQCP problems of the first step, a deterioration of both possessing time (over 10 fold increase, in most cases) and optimality gaps was observed (which settled around 1%, on average). Thirdly, also for $PF_k^{Lim} = 0$, the plateau for new generation allocation and maximum DG increased to 16 and 3.45, respectively. All of these indicate that the results returned by the solver should be more carefully validated. Regardless, in this particular test case, no bus has any baseline loads higher the new generators limits, therefore, given transmission and minimum generation boundaries are met, it should be expected that the solver will attempt to supply as much demand as possible locally, adding new resources in most or all candidate buses, which could yield substantially higher ***DG*** values.

Those preliminary results already hint that, although capable of dealing with the complex combinatorial AC OPF, the problems statement and formulation approaches, along with proper parametrizing, can substantially impact the solvers' performance and the results quality. Also, the introduction of precise transmission losses minimization (MIQCQP) seems to be substantially more unstable than finding overall maximum hosting capacity (MIQCP).

B. 118 bus case set up and results

The IEEE 118 bus standard case was also used. In this case, there are 53 PV buses (35 of which are based on synchronous compensators and the remaining on generating units), whereas 54 buses were set as candidates. Similar as before, the standard power flow baseline features were considered to tune the parameters applied, which were as following: all existing generators/compensators reactive power limits were set to be between -1.2pu and 1.2pu; generator active power limits were set as 0.8x to 1.2x the baseline case (including the slack bus), note that the limits in (27) are set to zero to model synchronous compensators; PFC equivalent limits in all buses were set to 0.2 (inductive or capacitive), except for transfer or PV-synchronous buses; minimum and maximum voltage magnitude limits are 0.94pu and 1.06pu, respectively, for all nodes, with voltage at controlled buses fixed at baseline values; and current magnitudes limits were set to 5.0pu in all branches. This time, new generation sites, when connected, must dispatch between 0.15pu and 0.75pu of active power and -0.45pu and 0.45pu of reactive power.

Analogous to the first test case, a series of (11)-(36) type problems are initially solved, considering increasingly higher values for the parameter NU^{Lim} (33), and $1.0 \leq$ ***DG*** ≤ 5.0. Fig. 3 summarizes those results.

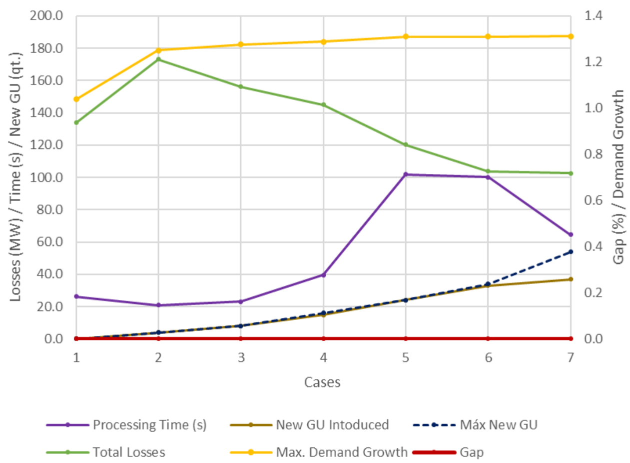

Fig. 3. ***DG*** maximizing in the 118 bus case

It can be observed form Fig. 3, that the overall profile of the results can elicit similar observations to the ones made regarding previous case (see Fig. 2). Total processing times fluctuated around 20s to 100s, whereas the maximum amount of new generator (even after NU^{Lim} = 54) reach a plateau at 37 new units, whereas the maximum ***DG*** stopped increasing with only 24 new connections, reaching up to 1.31. However, there is a marked diminishing return effect, since maximum ***DG*** already reaches approximately 1.29 with only 15 connections. Optimality gaps remained close from zero.

Also, it is important to highlight the non-linear increase in computational cost. That is, the 118bus case, with a few thousand variables and constraints, is only a few times larger the first problem, however, all processing times were consistently over 10 times higher, which suggests that this type of application could be unappealing or even intractable for larger networks (over thousands of buses), or multi-stage problems.

Once more, a follow up step was applied to minimize transmission losses through reallocation of the new generation sites introduced. Hence, similar to Table II, Table III compares resource allocation in each step. For consciences, original generators/compensator at the PV buses were omitted. Regarding the results in Table III, ***DG*** is fixed at 1.31x the baseline, whereas NU^{Lim} = 24 (i.e., the minimum amount to achieve the ***DG*** plateau in the first step problem). This yielded 24 new connections, totaling 92MW in transmission losses (the initial case was 133MW). Also, total processing time was 30s and the gap approximately 0.0%.

TABLE III
TRANSMISSION LOSS MINIMIZATION FOR THE 118 BUS CASE

Candidate Bus Nº (Initial)	P_g/Q_g (pu)	Candidate Bus Nº (Final)	P_g/Q_g (pu)
3 (*New*)	0.69/0.09	7 (*New*)	0.75/0.03
17 (*New*)	0.62/0.05	13 (*New*)	0.75/0.22
29 (*New*)	0.64/0.10	14 (*New*)	0.75/0.01
33 (*New*)	0.65/0.06	22 (*New*)	0.66/0.17
39 (*New*)	0.66/0.06	29 (*New*)	0.75/0.08
41 (*New*)	0.68/0.08	35 (*New*)	0.75/-0.02
47 (*New*)	0.25/0.24	39 (*New*)	0.75/0.10
48 (*New*)	0.61/0.22	41 (*New*)	0.75/0.12
50 (*New*)	0.22/0.05	45 (*New*)	0.75/0.35
51 (*New*)	0.75/-0.45	47 (*New*)	0.65/-0.06
53 (*New*)	0.75/0.05	52 (*New*)	0.63/0.13
57 (*New*)	0.15/0.00	53 (*New*)	0.75/0.15
58 (*New*)	0.15/0.00	57 (*New*)	0.66/0.05
60 (*New*)	0.75/0.45	60 (*New*)	0.75/0.45
67 (*New*)	0.15/-0.45	75 (*New*)	0.75/0.12
75 (*New*)	0.29/0.08	79 (*New*)	0.75/0.45
79 (*New*)	0.67/0.11	84 (*New*)	0.51/0.15
97 (*New*)	0.59/0.26	94 (*New*)	0.75/0.45
106 (*New*)	0.33/0.14	108 (*New*)	0.75/0.03
108 (*New*)	0.56/0.04	109 (*New*)	0.75/0.06
109 (*New*)	0.62/0.04	114 (*New*)	0.58/0.05
115 (*New*)	0.63/0.11	115 (*New*)	0.60/0.09
117 (*New*)	0.64/0.17	117 (*New*)	0.52/0.10
118 (*New*)	0.64/0.12	118 (*New*)	0.75/0.19

V. Conclusions

This work explored the application of advanced combinatorial optimization techniques using the Gurobi Optimizer to solve small to medium OPF problems. The study evaluated maximum generation capacity and transmission losses minimization in IEEE 30 and 118 bus standard case systems, demonstrating both the effectiveness and limitations of using commercial solvers in such contexts.

The results indicated that, although Gurobi can handle the underlying MIQCP and MIQCQP problems in the tested applications, the quality of solutions and computational performance are significantly influenced by the problem formulation and parameterization. The study showed that as generation limits and reactive power flow constraints are relaxed, the solver could achieve higher demand capacities and lower losses, albeit at the cost of increased processing time and, in some cases, higher optimality gaps. Moreover, a limited scalability potential was observed for larger systems, as the increase in processing time and solution complexity grew pronouncedly from the 30-bus to the 118-bus cases.

Therefore, this study contributes to the understanding of the potential and challenges of applying combinatorial optimization techniques in AC power systems, highlighting the importance of careful problem formulation and precise parameterization to achieve efficient and reliable results. Future research could explore hybrid methods or decomposition techniques to improve scalability and computational efficiency in larger networks.

VI. Acknowledgments

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VII. References

[1] M. Huneault and F. D. Galiana, "A survey of the optimal power flow literature," IEEE Trans. Power Syst., vol. 6, no. 2, pp. 762–770, May 1991, doi: 10.1109/59.76723.

[2] Carpentier, J., "Contribution to the economic dispatch problem.," Bulletin de la Societe Francoise des Electriciens, vol. 3, no. 8, pp. 431–447, 1962.

[3] H. Dommel and W. Tinney, "Optimal Power Flow Solutions," IEEE Trans. on Power Apparatus and Syst., vol. PAS-87, no. 10, pp. 1866–1876, Oct. 1968, doi: 10.1109/TPAS.1968.292150.

[4] M. Rayati, D. Teneketzis, "Electricity market design and implementation in the presence of asymmetrically informed strategic producers and consumers: A surrogate optimization-based mechanism," Energy Economics, vol. 109, pp. 105929, 2022, doi: https://doi.org/10.1016/j.eneco.2022.105929.

[5] L. A. Roald et al., "Power systems optimization under uncertainty: A review of methods and applications," Electric Power Systems Research, vol. 214, Part A, pp. 108725, 2023, doi: https://doi.org/10.1016/j.epsr.2022.108725.

[6] J. K. Skolfield and A. R. Escobedo, "Operations research in optimal power flow: A guide to recent and emerging methodologies and applications," European Journal of Operational Research, vol. 300, no. 2, pp. 387–404, Jul. 2022, doi: 10.1016/j.ejor.2021.10.003.

[7] H. Mataifa, S. Krishnamurthy and C. Kriger, "Volt/VAR Optimization: A Survey of Classical and Heuristic Optimization Methods," in IEEE Access, vol. 10, pp. 13379-13399, 2022, doi: 10.1109/ACCESS.2022.3146366.

[8] P. Charles, F. Mehazzem, and T. Soubdhan, "A Review on Optimal Power Flow Problems: Conventional and Metaheuristic Solutions," in 2020 2nd International Conference on Smart Power & Internet Energy Systems (SPIES), Bangkok, Thailand: IEEE, Sep. 2020, pp. 577–582. doi: 10.1109/SPIES48661.2020.9242994.

[9] B.-G. Risi, F. Riganti-Fulginei, and A. Laudani, "Modern Techniques for the Optimal Power Flow Problem: State of the Art," Energies, vol. 15, no. 17, p. 6387, Sep. 2022, doi: 10.3390/en15176387.

[10] A. M. Nassef et al., "Review of Metaheuristic Optimization Algorithms for Power Systems Problems," Sustainability, vol. 15, pp. 9434, 2023, doi: https://doi.org/10.3390/su15129434.

[11] Z. Wang, A. Younesi, M. V. Liu, G. C. Guo, and C. L. Anderson, "AC Optimal Power Flow in Power Systems With Renewable Energy Integration: A Review of Formulations and Case Studies," IEEE Access, vol. 11, pp. 102681–102712, 2023, doi: 10.1109/ACCESS.2023.3314330.

[12] V. N. Motta, M. F. Anjos, and M. Gendreau, "Survey of optimization models for power system operation and expansion planning with demand response," European Journal of Operational Research, vol. 312, no. 2, pp. 401–412, Jan. 2024, doi: 10.1016/j.ejor.2023.01.019.

[13] M. Khonji, S. C.-K. Chau, and K. Elbassioni, "Combinatorial Optimization of AC Optimal Power Flow With Discrete Demands in Radial Networks," IEEE Trans. Control Netw. Syst., vol. 7, no. 2, pp. 887–898, Jun. 2020, doi: 10.1109/TCNS.2019.2951657.

[14] C. Yang et al., "Optimal power flow in distribution network: A review on problem formulation and optimization methods," Energies, vol. 16, no. 16, pp. 5974, 2023, doi: https://doi.org/10.3390/en16165974.

[15] R. Moreno-Chuquen and S. Cantillo-Luna, "Assessment of a multiperiod optimal power flow for power system operation," Int. Rev. Electr. Eng., vol. 15, no. 6, pp. 484-492, 2020, doi: https://doi.org/10.15866/iree.v15i6.18304.

[16] M. Goerigk, M. Hartisch, "Combinatorial optimization problems with balanced regret," Discrete Applied Mathematics, vol. 328, pp. 40-59, 2023, doi: https://doi.org/10.1016/j.dam.2022.11.013.

[17] R. Anand, D. Aggarwal, and V. Kumar, "A comparative analysis of optimization solvers," Journal of Statistics and Management Systems, vol. 20, no. 4, pp. 623–635, Jul. 2017, doi: 10.1080/09720510.2017.1395182.

[18] S. Joglekar, S. Ellenrieder and M. Reuter-Oppermann, "Unlocking Solver Potential: A Framework for Analysis and Inter-Comparison of Optimisation Solvers," ACIS 2023 Proceedings, vol. 19, 2023.

[19] M. González, J. J. López-Espín, and J. Aparicio, "A Parallel Algorithm for Matheuristics: A Comparison of Optimization Solvers," Electronics, vol. 9, no. 9, p. 1541, Sep. 2020, doi: 10.3390/electronics9091541.

[20] "Decison Tree for Optimization Software." Accessed: Mar. 25, 2024. [Online]. Available: https://plato.asu.edu/bench.html.

[21] J. R. Donoghue, "Comparison of Integer Programming (IP) Solvers for Automated Test Assembly (ATA)," ETS Research Report Series, vol. 2015, no. 1, pp. 1–12, Jun. 2015, doi: 10.1002/ets2.12051.

[22] "MIQP: mixed integer programs with quadratic terms in the objective function," IBM. Accessed: Jul. 23, 2024. [Online]. Available: https://www.ibm.com/docs/en/icos/22.1.1?topic=smippqt-miqp-mixed-integer-programs-quadratic-terms-in-objective-function.

[23] "Gurobi Optimizer," Gurobi Optimization. Accessed: Mar. 25, 2024. [Online]. Available: https://www.gurobi.com/solutions/gurobi-optimizer/.

[24] A. S. Nair et al., "Computational and numerical analysis of AC optimal power flow formulations on large-scale power grids," Electric Power Systems Research, vol. 202, pp. 107594, 2022, doi: https://doi.org/10.1016/j.epsr.2021.107594.

[25] J. M. Home-Ortiz, W. C. De Oliveira and J. R. S. Mantovani, "Optimal Power Flow Problem Solution Through a Matheuristic Approach," in IEEE Access, vol. 9, pp. 84576-84587, 2021, doi: 10.1109/ACCESS.2021.3087626.

[26] O. D. Montoya, F. Zishan, D. A. Giral-Ramírez, "Recursive Convex Model for Optimal Power Flow Solution in Monopolar DC Networks," Mathematics, vol. 10, no. 19, pp. 3649, 2022, doi: https://doi.org/10.3390/math10193649.

[27] Y. Fan, L. Feng and G. Li, "Dynamic optimal power flow in distribution networks with wind/PV/storage based on second-order cone programming," 2020 5th Asia Conference on Power and Electrical Engineering (ACPEE), Chengdu, China, 2020, pp. 1136-1142, doi: 10.1109/ACPEE48638.2020.9136442.

[28] H. Wu, M. Wang, Z. Xu and Y. Jia, "Active Constraint Identification Assisted DC Optimal Power Flow," 2022 IEEE/IAS Industrial and Commercial Power System Asia (I&CPS Asia), Shanghai, China, 2022, pp. 185-189, doi: 10.1109/ICPSAsia55496.2022.9949655.

[29] S. E. Kayacık and B. Kocuk, "An MISOCP-Based Solution Approach to the Reactive Optimal Power Flow Problem," in IEEE Transactions on Power Systems, vol. 36, no. 1, pp. 529-532, Jan. 2021, doi: 10.1109/TPWRS.2020.3036235.

[30] H. Ahmadi and J. R. Martí, "Linear Current Flow Equations With Application to Distribution Systems Reconfiguration," in IEEE Transactions on Power Systems, vol. 30, no. 4, pp. 2073-2080, July 2015, doi: 10.1109/TPWRS.2014.2360363.

[31] L. Yang, E. S. Fraga and L. G. Papageorgiou, "Mathematical programming formulations for non-smooth and non-convex electricity dispatch problems," Electric Power Systems Research, vol. 95, pp. 302-308, 2013, doi: https://doi.org/10.1016/j.epsr.2012.09.015.

[32] D.P. Bohórquez-Álvarez, K.D. Niño-Perdomo and O.D. Montoya, "Optimal Load Redistribution in Distribution Systems Using a Mixed-Integer Convex Model Based on Electrical Momentum," Information, vol. 14, no. 4, pp. 229, 2023, doi: https://doi.org/10.3390/info14040229.

[33] J. Castañeda-Sanchez, E. Zurita-Márquez, D. Ortiz-Villalba, M. Saltos-Rodríguez, M. Aguirre-Velasco and A. Velásquez-Lozano, "Comparative Analysis of Linearized AC Power Flow Models for Power Distribution Systems," 2023 IEEE Seventh Ecuador Technical Chapters Meeting (ECTM), Ambato, Ecuador, 2023, pp. 1-6, doi: 10.1109/ETCM58927.2023.10308969.

[34] A. B. Neto et al., "Optimal current flow technique for distribution systems with renewable energy generation" Energy Systems, vol. 13, pp. 725–747, 2022, doi: https://doi.org/10.1007/s12667-020-00402-y.

[35] K. Youngmin el al., "Charging scheduling problem of an M-to-N electric vehicle charger," Applied Mathematical Modelling, vol. 64, pp. 603-614, 2018, doi: https://doi.org/10.1016/j.apm.2018.07.060.

[36] Z. Zhang et al., "Capacity optimal configuration and collaborative planning of multi-region integrated energy system," Energy, vol. 278, pp. 127970, Sep. 2023, doi: https://doi.org/10.1016/j.energy.2023.127970.

[37] A. B. Al Fahri, R. S. Wibowo and N. K. Aryani, "Optimal Power and Natural Gas Integrated System Considering Energy Storage Using Interior Point optimization (IPOPT)," 2022 International Seminar on Intelligent Technology and Its Applications (ISITIA), Surabaya, Indonesia, 2022, pp. 401–406, 2022, doi: 10.1109/TPWRS.2018.2874173.

[38] G. Qiu, M. Tanneau and P. V. Hentenryck, "Dual Conic Proxies for AC Optimal Power Flow," PSCC, 2024, doi: https://doi.org/10.48550/arXiv.2310.02969.

[39] "MIQCP: mixed integer programs with quadratic terms in the constraints," IBM. Accessed: Aug. 09, 2024. [Online]. Available: https://www.ibm.com/docs/en/cofz/12.10.0?topic=smippqt-miqcp-mixed-integer-programs-quadratic-terms-in-constraints.

[40] A. Lundell, J. Kronqvist and T. Westerlund, "The supporting hyperplane optimization toolkit for convex MINLP," Journal of Global Optimization, vol. 84, pp. 1–41, 2022, doi: https://doi.org/10.1007/s10898-022-01128-0.

[41] S. Cho, C. Li, and I. E. Grossmann, "Recent advances and challenges in optimization models for expansion planning of power systems and reliability optimization," Computers & Chemical Engineering, vol. 165, pp. 107924, 2022, doi: https://doi.org/10.1016/j.compchemeng.2022.107924.

[42] N. R. Watson and J. D. Watson, "An Overview of HVDC Technology," Energies, vol. 13, no. 17, pp. 4342, 2020, doi: https://doi.org/10.3390/en13174342.

[43] M. M. Ragab, R. A. Ibrahim, H. Desouki, et al., "Optimal energy management applying load elasticity integrating renewable resources," Scientific Reports, vol. 13, pp. 15000, 2023, doi: https://doi.org/10.1038/s41598-023-41929-1.

[44] "What's New – Gurobi 11.0," Gurobi Optimization. Accessed: Aug. 22, 2024. [Online]. Available: https://www.gurobi.com/whats-new-gurobi-11-0/.

[45] "Matpower 8.0," Matpower. Accessed: Aug. 22, 2024. [Online]. Available: https://matpower.org/matpower-8-0b1-launch/.

VIII. Biographies

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