A Constructive Heuristic Algorithm for Distribution Systems Expansion Planning Considering Reliability

1st Lluleisi Grández
Department of Electrical Engineering
School of Engineering
São Paulo State University (UNESP)
Ilha Solteira, Brazil
lluleisi.grandez@unesp.br

2nd Leonardo H. Macedo
Department of Engineering
School of Engineering and Sciences
São Paulo State University (UNESP)
Rosana, Brazil
leohfmp@ieee.org

3rd Rubén Romero
Department of Electrical Engineering
School of Engineering
São Paulo State University (UNESP)
Ilha Solteira, Brazil
ruben.romero@unesp.br

***Abstract*—The distribution systems expansion planning problem is a relevant and highly complex topic in the field of optimizing the operation and planning of electrical energy systems. The mathematical model of this problem is a mixed-integer nonlinear programming problem with combinatorial characteristics. When reliability requirements are considered, this optimization problem becomes more complex. There are different ways to consider reliability when planning the expansion of electrical energy distribution systems. In this work, an alternative approach is proposed. Traditionally, distribution systems are expanded taking into account specified reliability indices. In the proposal presented, reliability guarantees the greatest topological diversity of the system, which, therefore, can present high-reliability indices for specified indices and other unspecified performance criteria. Test results are shown using data from a 54-bus system.**

***Index Terms*—Distribution systems expansion planning, mixed-integer programming, optimization of electrical systems, radial topology, reliability.**

I. Introduction

The primary distribution systems expansion planning (DSEP) problem is a highly relevant topic for the operation and optimal planning of the distribution system. The primary distribution network refers to the electrical network that starts at the substation and ends at the low-voltage transformers. From these transformers, the secondary distribution network supplies residential and commercial loads.

In the DSEP problem, given the current network, demand data for a planning horizon, and the possibilities of constructing new substations, reinforcing existing substations, replacing existing conductors, and constructing new conductors in new branches, the goal is to expand the system at the lowest cost while ensuring that the expanded system operates adequately for the new demand values of the planning horizon. The optimal solution should specify the new substations to be built, the substations to be reinforced, the conductors to be replaced, and the new conductors to be constructed in new branches, detailing the characteristics of each component to be built.

A typical feature of the DSEP problem is that the system must operate in a radial topology for various reasons. This is an old paradigm that remains valid today. Until recently, the DSEP problem was expanded by finding only one final topology. However, in practice, a distribution system is built weakly meshed but, for safety reasons, operates radially. The strategy of finding an optimal radial topology presented two problems in the specialized literature: (1) it was not possible to find a complete mathematical model because the radial topology requirement could not be represented as simple algebraic constraints, and (2) the proposal to find an optimal radial topology did not align with the logic of a distribution system that should be built in a meshed manner but operate in a radial topology. The first problem was solved in the past decade with various formulations to represent the radial topology requirement using simple algebraic relationships, as presented in [1], [2]. The other issue represents a current and relevant research topic addressed in this work.

Many publications find only an expansion plan that is merely an optimal radial expansion topology. These publications have used almost all existing optimization techniques from the specialized literature. Such proposals are presented in [3]–[7]. The proposal in [3] presents a genetic algorithm, while [4] offers a mathematical model used as a basis for more complex problems, and [5] introduces a sophisticated constructive heuristic algorithm. On the other hand, [6], [7] solve the DSEP problem using complete models with the radial topology constraints incorporated into the mathematical model and commercial optimization solvers.

As mentioned earlier, current relevant research on the DSEP problem involves finding an optimal expansion plan in a meshed topology. In this context, the optimization strategy should indicate the radial topology for normal operating conditions and the constructed branches that should remain open during normal operation but can be used for abnormal conditions, such as during maintenance or in case of faults when some system branches need to be deactivated for safety. In this context, besides finding an optimal expansion plan in terms of expansion costs, the expanded network must meet reliability criteria.

Two different strategies exist to expand the distribution system under this new paradigm. The first strategy involves expanding the distribution system in two consecutive phases.

This work was supported by the Coordination for the Improvement of Higher Education Personnel (CAPES) – Finance Code 001 and by the Brazilian National Council for Scientific and Technological Development (CNPq), under grants 306180/2022-7 and 409062/2023-5.

In the first phase, the radial topology is found using the same strategies as the previous paradigm, i.e., the traditional method used in references [3]–[5]. In the second phase, extra branches are added using some reliability-related logic. Obviously, this strategy cannot find optimal expansion plans because it separates a problem that should be optimized integrally into two consecutive phases. Therefore, this strategy is used when, for some reason, it is impossible to solve the problem integrally. Hence, this strategy is used when no known mathematical model exists or when the known mathematical model cannot be solved for complex instances using efficient optimization solvers. The second strategy involves expanding the distribution system integrally, considering expansion costs, and incorporating reliability requirements, usually adding reliability constraints based on indices used to measure the distribution system's reliability.

In [8], there is concern about considering reliability. However, the system is expanded in the traditional way, and subsequently, the radial topology with the highest reliability is analyzed. The proposals in [9], [10] show concern for expansion with reliability, preceding more formal proposals that consider reliability as part of the expansion process. The proposals in [11]–[17] represent the most relevant proposals, according to the authors, for the second optimization approach of the DSEP problem, i.e., finding the optimal expansion plan integrally while considering expansion costs and incorporating reliability requirements in the form of constraints to satisfy different reliability indices. Notably, the proposals presented in [12], [17] should be highlighted. Regarding these proposals, it should be mentioned that the fundamental idea is to find a highly complex mathematical model and solve it using an optimization solver. Thus, each reliability index can be incorporated into the mathematical model as constraints.

The optimization proposal presented in this work follows the second optimization strategy, i.e., it finds an expanded meshed network indicating the branches that should operate in a radial topology during normal operations and the branches that should remain open but can operate by reconfiguring the operating topology in atypical cases (during maintenance or permanent faults in the distribution system). Therefore, the difference between this proposal and those presented in [11]–[17] is the way of conceptualizing the reliability of the expanded system. This proposal assumes that an expanded distribution network generally adequately addresses system faults if this expanded meshed system has the highest number of radial topologies that can be generated from this meshed topology. For illustration, suppose there is a 50-bus system, and the expansion proposal has 54 constructed branches. In this context, the system operates in a radial topology with 49 closed and five open branches. There is a connected graph with 54 branches for such an expansion plan, and it is possible to find the number of different radial topologies in this graph. Thus, if there are two expansion plans, Plan A and Plan B, with 54 constructed branches, then the plan with the highest number of different radial topologies has higher reliability. This observation, initially empirical but consistent, is due to the fact that if a fault occurs that deactivates a branch or a set of branches, the plan with the highest number of radial topologies has a greater chance of keeping the system connected through a radial topology.

The suggested proposal is feasible only with the relatively recent discovery that it is possible to determine the exact number of radial topologies in a meshed distribution system. In other words, for a connected meshed graph, it is possible to calculate the number of spanning trees (radial topologies) in that graph [18], [19]. It should be noted that this proposal prioritizes topological robustness over a specific reliability index. Thus, the proposal is conceptually very different from those presented in [2]–[17]. Currently, no mathematical model incorporates this requirement of the DSEP problem. For this reason, the optimization proposal consists of two phases: in the first phase, the traditional mathematical model is solved to find the optimal radial operating topology, and in the second phase, a set of branches is added to find a meshed expanded system with the highest number of different radial topologies.

In summary, this work finds an optimal radial expansion topology using the traditional mathematical model for this problem and then adds a specified set of extra branches. These branches are chosen so the resulting graph has the highest possible number of radial topologies. This second phase is solved using a constructive heuristic algorithm.

II. Mathematical Model

A mixed-integer second-order cone programming (MISOCP) mathematical model for the DSEP problem is presented, which finds the optimal radial operating topology for an expanded distribution system to meet the demand of a planning horizon. This allows for constructing new substations, reinforcing existing substations, replacing existing conductors, and constructing conductors in new branches, as shown in (1)–(13).

$$\text{minimize } v = \sum_{i \in \Omega_s} C_i^s w_i + \sum_{ij \in \Omega_c} \sum_{a \in \Omega_a} C_{t_{ij,a}}^{ra} l_{ij} y_{ij,a} \quad (1)$$

Subject to:

$$\sum_{ji \in \Omega_c} \sum_{a \in \Omega_a} P_{ji,a} - \sum_{ij \in \Omega_c} \sum_{a \in \Omega_a} \left(P_{ij,a} + R_a l_{ij} I_{ij,a}^{sq} \right) + P_i^S = P_i^D \quad (2)$$

$$\sum_{ji \in \Omega_c} \sum_{a \in \Omega_a} Q_{ji,a} - \sum_{ij \in \Omega_c} \sum_{a \in \Omega_a} \left(Q_{ij,a} + X_a l_{ij} I_{ij,a}^{sq} \right) + Q_i^S = Q_i^D \quad (3)$$

$$\forall i \in \Omega_b$$

$$V_i^{sq} - V_j^{sq} = \sum_{a \in \Omega_a} \left[2 \left(R_a P_{ij,a} + Q_{ij,a} \right) + Z_a^2 l_{ij}^2 I_{ij,a}^{sq} \right] + b_{ij} \quad (4)$$

$$\forall ij \in \Omega_c$$

$$V_j^{sq} I_{ij,a}^{sq} \geq P_{ij,a}^2 + Q_{ij,a}^2 \quad (5)$$
$$\forall ij \in \Omega_c, a \in \Omega_a$$

$$|b_{ij}| \leq \left(\overline{V}^2 - \underline{V}^2\right)\left(1 - \sum_{a \in \Omega_a} y_{ij,a}\right) \quad (6)$$
$$\forall ij \in \Omega_c$$

$$\underline{V}^2 \leq V_i^{sq} \leq \overline{V}^2 \quad (7)$$
$$\forall i \in \Omega_b$$

$$0 \leq I_{ij,a}^{sq} \leq \overline{I}_a^2 y_{ij,a} \quad (8)$$
$$\forall ij \in \Omega_c, a \in \Omega_a$$

$$\sum_{a \in \Omega_a} y_{ij,a} \leq 1 \quad (9)$$
$$\forall ij \in \Omega_c$$

$$\sum_{ij \in \Omega_c} \sum_{a \in \Omega_a} y_{ij,a} = |\Omega_b| - |\Omega_s| \quad (10)$$

$$\left(P_i^S\right)^2 + \left(Q_i^S\right)^2 \leq \overline{S}_i^2 + S_i^{gn}\left(\overline{S}_i + S_i^{gn}\right) w_i \quad (11)$$
$$\forall i \in \Omega_s$$

$$w_i \in \{0,1\} \quad (12)$$
$$\forall i \in \Omega_s$$

$$y_{ij,a} \in \{0,1\} \quad (13)$$
$$\forall ij \in \Omega_c, a \in \Omega_a$$

The model includes various types of quantities:

- *Sets:* Ω_b is the set of buses, Ω_s is the set of substation buses, Ω_c is the set of branches, and Ω_a is the set of conductor types that can be installed on each branch.
- *Parameters:* R_a, X_a, and Z_a are the resistance, reactance, and magnitude of the impedance per km of conductor type a, respectively, l_{ij} is the length of branch ij, P_i^D and Q_i^D are the active and reactive power demanded at bus i, C_i^s is the construction or expansion cost of a substation at bus i, $C_{t_{ij,a}}^{ra}$ is the cost per km of conductor type a, $\overline{S}_i$ is the capacity of a substation at bus i, S_i^{gn} is the capacity of a reinforcement substation at bus i, $\underline{V}$ and $\overline{V}$ are the lower and upper limits of the voltage magnitude at system buses, t_{ij} is the type of existing conductor on branch ij, and $\overline{I}_a$ is the current magnitude limit of conductor type a.
- *Variables:* V_i^{sq} is the square of the voltage magnitude at bus i, b_{ij} is an auxiliary variable that controls the voltage drop in a branch when the branch is operating or not, P_i^S and Q_i^S are the active and reactive power injected by the substation at bus i, $I_{ij,a}^{sq}$ is the square of the current magnitude on branch ij with conductor type a, $P_{ij,a}$ and $Q_{ij,a}$ are the active and reactive power on branch ij for conductor type a, $y_{ij,a}$ is a binary variable that equals $y_{ij,a} = 1$ if a conductor type a is constructed on branch ij and zero otherwise, and w_i is a binary variable that equals $w_i = 1$ if a substation is constructed or reinforced at bus i and zero otherwise.

The first part of the objective function (1) represents the investment cost in substations, and the second term corresponds to the construction and installation costs of conductors, i.e., the investment costs in installing and replacing conductors. Constraints (2) and (3) represent the active and reactive power balance at each bus of the electrical system, constraints (4) and (5) represent constraints resulting from the application of Kirchhoff's voltage law in the electrical system, constraint (6) controls the voltage at the ends of an electrical branch ij using the auxiliary variable b_{ij} such that if $b_{ij} = 0$ the branch ij is constructed and therefore the voltage at the ends of the branch ij must comply with Kirchhoff's voltage law, otherwise it should be relaxed. Constraints (7) and (8) represent the operational limits of the voltage magnitude at each bus and the current at each branch, respectively, constraint (9) allows selecting only one type of conductor when the branch ij is constructed or operating. Constraint (10) ensures that the system is expanded to have a number of branches equal to the number of buses minus the number of substations. This condition, together with constraint (2), ensures that the solution found must be radial. Here, $|\Omega_b|$ is the number of buses in the system, and $|\Omega_s|$ is the number of substations in the electrical system. Constraint (11) ensures that the power supplied by each substation respects the capacity of the respective substation. Finally, (12) and (13) indicate the binary nature of the investment variables w_i and $y_{ij,a}$. The mathematical model of the DSEP problem can be solved using an efficient commercial solver.

III. Expansion with Reliability

The proposal of this work involves two phases, as mentioned earlier. In the first phase, an optimal radial operating topology is found using the mathematical model from the previous section. In the second phase, a set of p extra branches must be added to increase the system's reliability. The system should operate with the radial topology found in the first phase under normal operating conditions, and the p extra branches should remain disconnected. However, under atypical conditions such as permanent faults, the system should be reconfigured with the p branches that can be connected.

An initial proposal is presented to find a weakly meshed topology from a radial topology to increase the reliability of an expanded distribution system. The central idea is to add p extra branches to the previously found topology so that the weakly meshed system presents the highest number of radial topologies in the corresponding graph of this distribution system. The input data for this problem are the radial topology found using classical optimization and a set of ns candidate branches for addition, from which p branches must be chosen so that the expanded system is a weakly meshed system with the highest number of existing radial topologies in the graph of this system.

The aforementioned problem can be solved using classical optimization, a heuristic, or a metaheuristic. In this work, a constructive heuristic algorithm (CHA) was devised. A CHA finds a solution, usually of good quality, to a complex problem through an iterative process where a component of the solution being constructed is added at each step. The most critical decision in a CHA is the choice or design of a sensitivity indicator that identifies the component that should be added to the solution under construction. Thus, a CHA is simple to implement and is generally very fast.

For the second phase of the DSEP problem, the implemented CHA takes the following form:

1) Store the radial topology found in phase 1, which becomes the current topology. Store the ns candidate branches for addition, called free branches, and choose the value of p (the number of branches to be added).
2) Check if p branches have already been added to the current topology. If p branches have been added, then stop the optimization process. Otherwise, go to Step 3.
3) Find the sensitivity indicator values for each free branch.
4) With the sensitivity indicators found in the previous step, identify the free branch that should be incorporated into the current topology. Add this branch to the current topology and update the set of free branches and added branches. Return to Step 2.

In the CHA shown above, only the sensitivity indicator must be designed. Thus, the designed sensitivity indicator is shown in (14).

$$IS_{ij} = ntr_{ij} \tag{14}$$

where ntr_{ij} is the number of radial topologies existing in the graph of the current topology with the addition of the free branch ij. Therefore, the free branch ij with the highest ntr_{ij} value should be chosen to be incorporated into the current topology.

An existing concept in graph theory is used to find the number of radial topologies existing in the meshed graph of a distribution system. In [18], the related theory is presented and analyzed in [19] for the reconfiguration problem. It is a new paradigm that is still rarely used in optimizing distribution systems that require a radial topology for operation. The central issue is that it is possible to find the exact number of radial topologies in a connected graph (a connected distribution system [18]) in the form of the Matrix-Tree Theorem.

Matrix-Tree Theorem: Let G be a connected graph and L be the Laplacian matrix of this graph. The number of spanning trees (radial topologies) of this graph G equals the value of any cofactor L_{ij} of L.

The previous theorem allows finding the exact number of radial topologies in a weakly meshed distribution system. To do this, the Laplacian matrix L must be constructed, where each matrix element has the structure shown in (15).

$$l_{ij} = \begin{cases} \text{degree}(i) & \text{If } i = j \\ -1 & \text{If there is a branch between } i \text{ e } j \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

To find the number of radial topologies ntr, the modified Laplacian matrix (LM) must be found by eliminating a row and a column from the matrix L, which will be used in (16)

$$ntr_{ij} = |\det(LM)| \tag{16}$$

Therefore, in the CHA, for each new candidate graph found with the addition of a new branch, it is possible to find the number of radial topologies ntr_{ij} by calculating the determinant of a matrix. Thus, for each free branch in Step 3 of the CHA, the value of ntr_{ij} of the current graph with the addition of the free branch ij must be found. In Step 4 of the CHA, the free branch with the highest value of ntr_{ij} is chosen for addition.

IV. Tests and Results

The expansion proposal was applied to the 54-bus system, whose base topology is shown in Figure 1, and whose complete data can be found in [5]. The CPLEX solver was used to solve the model written in AMPL. The system has 50 demand buses and four substations, with substations at buses 101 and 102 in the base topology having a capacity of 16.7 MVA and a nominal voltage of 13.5 kV. These buses can be reinforced with the same capacity. New substations with a nominal capacity of 22 MVA can be built at buses 103 and 104. The voltage limits at the buses are 0.95 and 1.0 p.u.

Table I shows the data of the base topology of the system's branches, where both the existing branches in the base topology and the new branches are listed. In the first phase, the mathematical model found the optimal radial topology. This radial topology is shown in Figure 2 and Table II. A substation was also added at bus 103 and another at bus 104, each with a nominal capacity of 22 MVA. No reinforcements were added to the substations at buses 101 and 102. In the optimization process, three branches were reconductored from among the existing conductors in the base topology, while one of them was disconnected.

The expansion cost in the first phase was v = 4,788,328.00 USD, with 2 MUSD for the installation of the substation at bus 103, 2.4 MUSD for the installation of the substation at bus

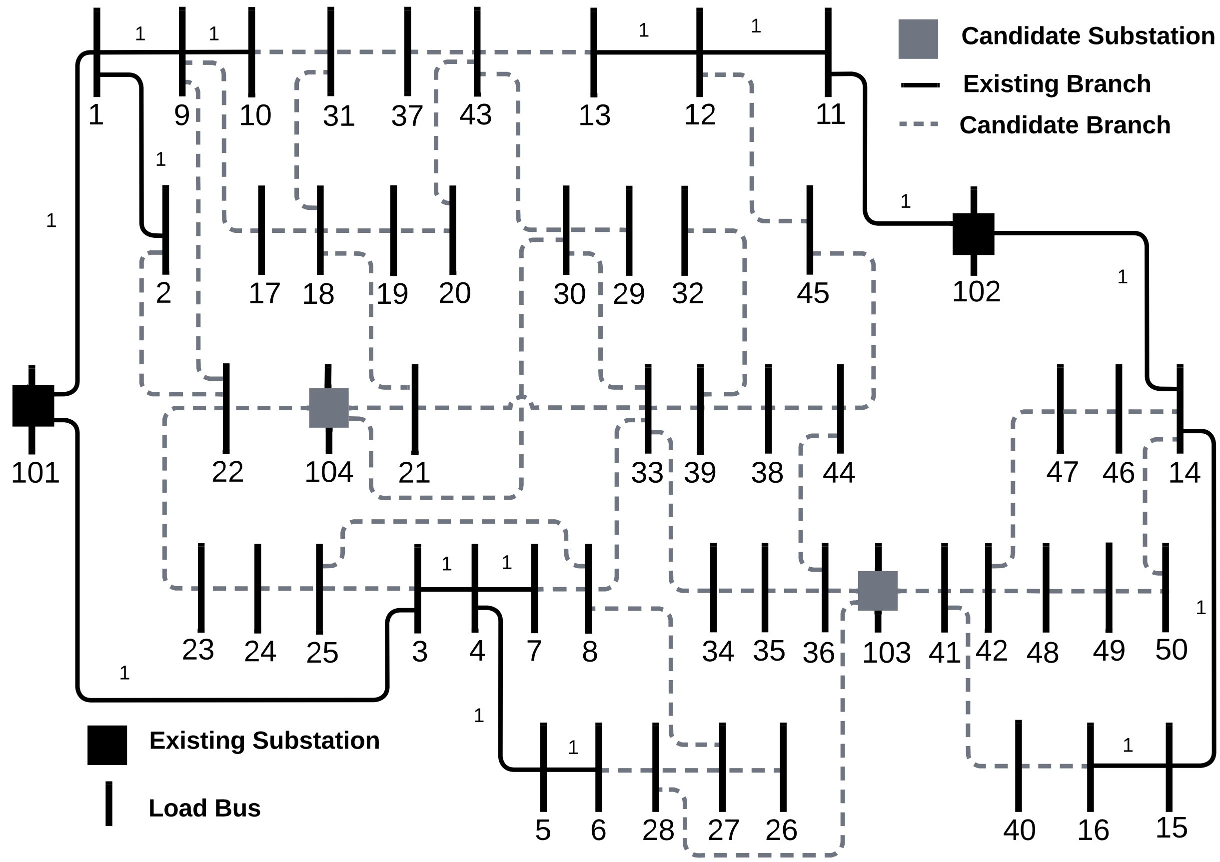

Fig. 1. Base topology.

104, and 388.328 kUSD for the installation of new conductors and reconductoring existing ones.

To implement the second phase, the optimization process must start from the radial topology found in the first phase. An additional set of branches specified among those not added in the first phase must be added. Thus, it was chosen to add six extra branches from the 19 available branches. The CHA added the branches according to the sequence shown in Table III. This table shows the sequence in which each branch is added, starting from the radial topology found in the first phase, and ntr indicates the number of existing radial topologies in the current expansion proposal. Therefore, the expanded system should operate with the radial topology found in the first phase, and the six branches shown in Table IV should also be constructed. These branches can operate under atypical operating conditions to reconfigure the operating topology, thus providing reliability to the expanded system. Therefore, the expanded system can operate with 135,877 different radial topologies.

Finally, the type of conductor must be chosen for the six branches selected in the second phase. An initial empirical strategy implemented in this work is to select the type of conductor considering the highest capacity conductor connected to one of the buses of the added branch. For example, branch 43, which connects buses 39 and 38, should be of type 1 since a type 1 conductor connects bus 38. This information was incorporated into Table IV. The cost of expansion by adding the six additional branches is 58,050.00 USD.

V. Conclusion

In this article, a new strategy was devised to solve the DSEP problem, considering the reliability of the distribution system by increasing the system's reconfiguration capability when the system graph can generate the highest number of radial topologies. This proposal differs from the one where reliability indices are incorporated into the optimization mathematical model, making the model difficult to solve. The results found are promising. The presented proposal is inspired by the possibility of knowing the number of radial topologies in a connected graph. As this is a first work, many topics remain to be explored. Therefore, future works should address topics such as a comparative analysis with other optimization proposals aiming at performance in terms of reliability, testing large and highly complex systems by solving the second phase with a metaheuristic, solving the second phase using a mathematical model, which means finding this mathematical model, solving the integrated problem using metaheuristics, solving the integrated problem by finding a mathematical

TABLE I
BRANCH DATA FOR THE 54-BUS SYSTEM.

Branch number	i	j	Type of conductor	Branch number	i	j	Type of conductor	Branch number	i	j	Type of conductor
1	1	101	1	24	22	9	0	47	34	33	0
2	3	101	1	25	23	22	0	48	35	34	0
3	4	3	1	26	24	23	0	49	36	35	0
4	7	4	1	27	25	24	0	50	36	103	0
5	5	4	1	28	8	25	0	51	28	103	0
6	8	7	0	29	27	8	0	52	41	103	0
7	6	5	1	30	26	27	0	53	40	41	0
8	9	1	1	31	28	27	0	54	16	40	0
9	2	1	1	32	28	6	0	55	42	41	0
10	10	9	1	33	30	104	0	56	48	42	0
11	14	102	1	34	29	30	0	57	49	48	0
12	15	14	1	35	43	30	0	58	50	49	0
13	16	15	1	36	37	43	0	59	47	42	0
14	11	102	1	37	31	37	0	60	46	47	0
15	12	11	1	38	10	31	0	61	14	46	0
16	13	12	1	39	43	13	0	62	18	31	0
17	20	19	0	40	45	12	0	63	20	43	0
18	19	18	0	41	44	45	0	64	2	22	0
19	18	17	0	42	38	44	0	65	30	33	0
20	17	9	0	43	39	38	0	66	21	33	0
21	21	18	0	44	32	39	0	67	36	44	0
22	21	104	0	45	33	39	0	68	25	3	0
23	22	104	0	46	8	33	0	69	50	14	0

TABLE II
BRANCHES OF THE RADIAL TOPOLOGY.

Branch number	i	j	Type of conductor	Cost (USD)	Branch number	i	j	Type of conductor	Cost (USD)	Branch number	i	j	Type of conductor	Cost (USD)
1	1	101	4	12364	21	21	18	3	13104	48	35	34	1	6540
2	3	101	1	0	22	21	104	4	11500	49	36	35	1	6540
3	4	3	1	0	23	22	104	1	11250	50	36	103	4	11500
4	7	4	1	0	25	23	22	1	10290	51	28	103	4	14352
7	6	5	1	0	26	24	23	1	8430	52	41	103	1	9360
8	9	1	2	11662	28	8	25	1	8430	53	40	41	1	11250
9	2	1	1	0	29	27	8	1	11250	56	48	42	1	7500
10	10	9	1	0	30	26	27	1	10290	57	49	48	1	11250
11	14	102	4	16500	31	28	27	2	10920	58	50	49	1	6540
12	15	14	1	0	32	28	6	1	15000	60	46	47	1	9360
13	16	15	1	0	33	30	104	4	12926	61	14	46	1	10290
14	11	102	1	0	34	29	30	1	9360	62	18	31	1	6540
15	12	11	1	0	37	31	37	1	5610	63	20	43	1	9360
16	13	12	1	0	41	44	45	1	6540	65	30	33	2	8750
17	20	19	1	9360	42	38	44	1	9360	67	36	44	1	8430
18	19	18	1	7500	44	32	39	1	12180	69	50	14	1	5610
20	17	9	1	12900	45	33	39	1	8430					

TABLE III
SEQUENCE OF BRANCHES ADDED.

	Iteration					
	1	2	3	4	5	6
Branch	39 (43-13)	27 (25-24)	55 (42-41)	43 (39-38)	38 (10-31)	5 (5-4)
ntr	9	72	504	3528	23128	135877

TABLE IV
BRANCHES ADDED IN PHASE 2.

Branch number	i	j	State	Type of conductor	Branch number	i	j	State	Type of conductor
5	5	4	1	1	39	43	13	1	1
27	25	24	1	1	43	39	38	1	1
38	10	31	1	1	55	42	41	1	1

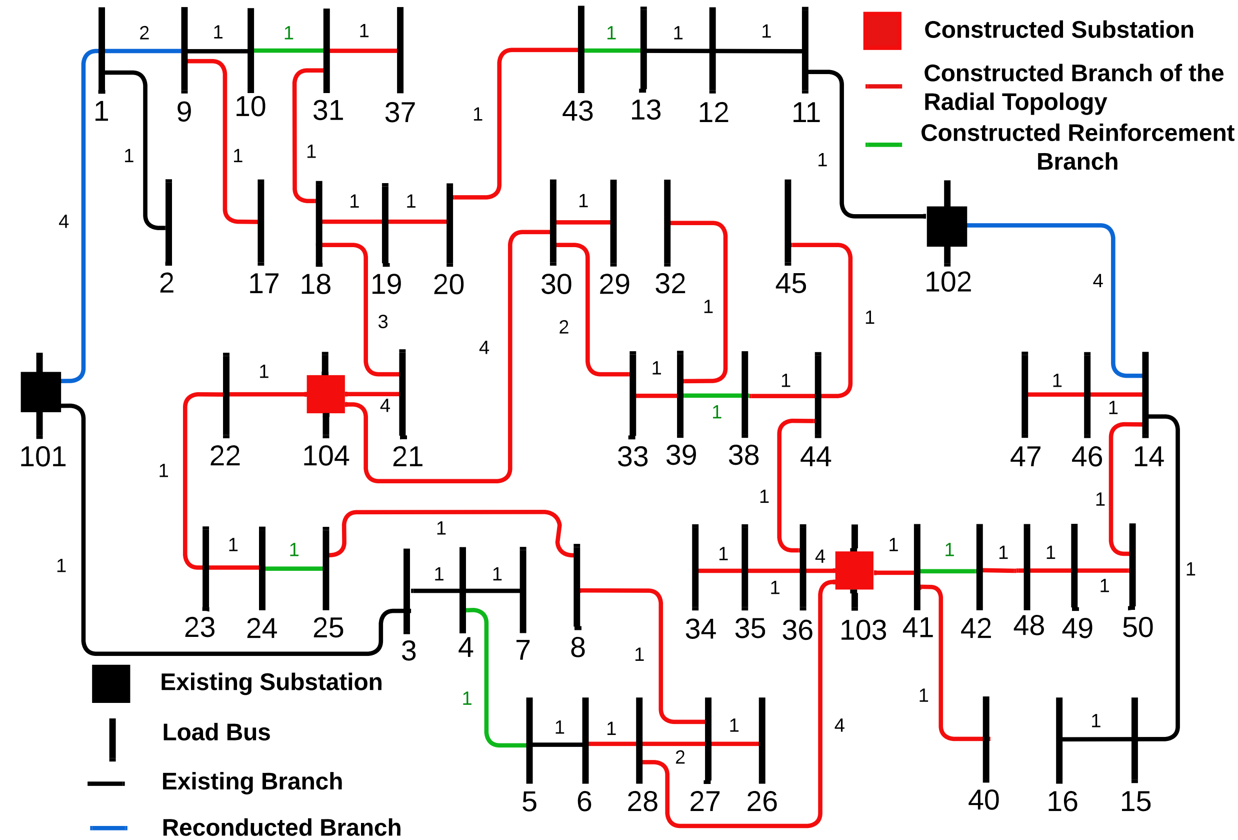

Fig. 2. Final topology.

model for this type of problem and solving this model using efficient solvers, among others.

REFERENCES

[1] M. Lavorato, J. F. Franco, M. J. Rider, and R. Romero, "Imposing radiality constraints in distribution system optimization problems," *IEEE Transactions on Power Systems*, vol. 27, no. 1, p. 172–180, Feb. 2012. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2011.2161349

[2] R. A. Jabr, R. Singh, and B. C. Pal, "Minimum loss network reconfiguration using mixed-integer convex programming," *IEEE Transactions on Power Systems*, vol. 27, no. 2, p. 1106–1115, May 2012. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2011.2180406

[3] V. Miranda, J. Ranito, and L. Proenca, "Genetic algorithms in optimal multistage distribution network planning," *IEEE Transactions on Power Systems*, vol. 9, no. 4, p. 1927–1933, 1994. [Online]. Available: http://dx.doi.org/10.1109/59.331452

[4] S. Haffner, L. Pereira, L. Pereira, and L. Barreto, "Multistage model for distribution expansion planning with distributed generation—Part I: Problem formulation," *IEEE Transactions on Power Delivery*, vol. 23, no. 2, p. 915–923, Apr. 2008. [Online]. Available: http://dx.doi.org/10.1109/TPWRD.2008.917916

[5] M. Lavorato, M. J. Rider, A. V. Garcia, and R. Romero, "A constructive heuristic algorithm for distribution system planning," *IEEE Transactions on Power Systems*, vol. 25, no. 3, p. 1734–1742, Aug. 2010. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2009.2038164

[6] J. F. Franco, M. J. Rider, M. Lavorato, and R. Romero, "Optimal conductor size selection and reconductoring in radial distribution systems using a mixed-integer LP approach," *IEEE Transactions on Power Systems*, vol. 28, no. 1, p. 10–20, Feb. 2013. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2012.2201263

[7] J. F. Franco, M. J. Rider, and R. Romero, "A mixed-integer quadratically-constrained programming model for the distribution system expansion planning," *International Journal of Electrical Power & Energy Systems*, vol. 62, p. 265–272, Nov. 2014. [Online]. Available: http://dx.doi.org/10.1016/j.ijepes.2014.04.048

[8] R. C. Lotero and J. Contreras, "Distribution system planning with reliability," *IEEE Transactions on Power Delivery*, vol. 26, no. 4, p. 2552–2562, Oct. 2011. [Online]. Available: http://dx.doi.org/10.1109/TPWRD.2011.2167990

[9] A. Tabares, J. F. Franco, M. Lavorato, and M. J. Rider, "Multistage long-term expansion planning of electrical distribution systems considering multiple alternatives," *IEEE Transactions on Power Systems*, vol. 31, no. 3, p. 1900–1914, May 2016. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2015.2448942

[10] N. C. Koutsoukis, P. S. Georgilakis, and N. D. Hatziargyriou, "Multistage coordinated planning of active distribution networks," *IEEE Transactions on Power Systems*, vol. 33, no. 1, p. 32–44, Jan. 2018. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2017.2699696

[11] G. Muñoz-Delgado, J. Contreras, and J. M. Arroyo, "Reliability assessment for distribution optimization models: A non-simulation-based linear programming approach," in *2017 IEEE Power &; Energy Society General Meeting*. IEEE, Jul. 2017. [Online]. Available: http://dx.doi.org/10.1109/PESGM.2017.8274173

[12] ——, "Distribution network expansion planning with an explicit formulation for reliability assessment," *IEEE Transactions on Power Systems*, vol. 33, no. 3, p. 2583–2596, May 2018. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2017.2764331

[13] A. Tabares, G. Muñoz-Delgado, J. F. Franco, J. M. Arroyo, and J. Contreras, "An enhanced algebraic approach for the analytical reliability assessment of distribution systems," *IEEE Transactions on Power Systems*, vol. 34, no. 4, p. 2870–2879, Jul. 2019. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2019.2892507

[14] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, H. Farzin, M. Moeini-Aghtaie, and M. Lehtonen, "A MILP model for incorporating reliability indices in distribution system expansion planning," *IEEE Transactions on Power Systems*, vol. 34, no. 3, p. 2453–2456, May 2019. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2019.2892625

[15] M. Jooshaki, A. Abbaspour, M. Fotuhi-Firuzabad, M. Moeini-Aghtaie, and M. Lehtonen, "MILP model of electricity distribution system expansion planning considering incentive reliability regulations," *IEEE Transactions on Power Systems*, vol. 34, no. 6, p. 4300–4316, Nov. 2019. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2019.2914516

[16] Z. Li, W. Wu, B. Zhang, and X. Tai, "Analytical reliability assessment method for complex distribution networks considering post-fault network reconfiguration," *IEEE Transactions on Power Systems*, vol. 35, no. 2, p. 1457–1467, Mar. 2020. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2019.2936543

[17] Z. Li, W. Wu, X. Tai, and B. Zhang, "A reliability-constrained expansion planning model for mesh distribution networks," *IEEE Transactions on Power Systems*, vol. 36, no. 2, p. 948–960, Mar. 2021. [Online]. Available: http://dx.doi.org/10.1109/TPWRS.2020.3015061

[18] J. Harris, J. L. Hirst, and M. Mossinghoff, *Combinatorics and Graph Theory*. Springer New York, 2008. [Online]. Available: http://dx.doi.org/10.1007/978-0-387-79711-3

[19] L. H. Macedo, J. F. Franco, M. Mahdavi, and R. Romero, "A contribution to the optimization of the reconfiguration problem in radial distribution systems," *Journal of Control, Automation and Electrical Systems*, vol. 29, no. 6, p. 756–768, Sep. 2018. [Online]. Available: http://dx.doi.org/10.1007/s40313-018-0415-6